boundary of the high conducting layer. As the velocity increases this charged layer is partly removed by the wind so that the electric field can penetrate the highly ionized region and increase the effectiveness of charge separation. The charge in the shielding layer is carried away by the wind and an equal and opposite charge is carried to the ionizer by the action of the field on ions in the ionized layer.

An analysis for the physical model just described which calculates the current generated by the removal of the shielding layer charge as a function of wind velocity and externally imposed field has been worked out (Hill and Hoppel[3]) and will be presented at the Fifth International Conference on Atmospheric Electricity as part of a paper[4] on radioactive collectors. Here we cite only the equation for the current per unit width without derivation.

$$\frac{i}{w} = \epsilon \frac{\Delta\lambda}{\lambda(z_o)} V_x E \left[\exp\left(-\frac{\lambda(z_o)x}{\epsilon V_x}\right) - 1\right]$$

where

$$E = E(\infty)\exp\left\{0.34\frac{\Delta\lambda}{\lambda(z_o)}\left[\exp\left(-\frac{\lambda(z_o)x}{\epsilon V_x}\right) - 1\right]\right\}$$

where

V_x is the velocity aspirating the ionizer,

$E(\infty)$ is the perpendicular component of the electric field far enough from the collector to be uninfluenced by the shielding charge,

$\Delta\lambda$ is the change in conductivity across the boundary of the highly ionized layer and is determined from the ionization rate and recombination coefficient.

$\lambda(z_o)$ is the conductivity at the boundary and here taken to be half of $\Delta\lambda$.

x is the length of the ionizer.

ϵ is the permittivity of free space.

Equation (1) has been verified experimentally as shown in Fig. 1 which compares currents generated by an ionizer in a wind tunnel as a function of velocity with those predicted by Eq. (1) for different values of electric field.

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Reply by Author to M. L. Hill and W. A. Hoppel

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I WOULD like to thank Messrs. Hill and Hoppel for their comments on my paper and also for providing me with a more detailed version of their theory which was of assistance in preparing this response.

In reply to the comment that the ionizers are insulated, I would like to state that, within the context of the model, the electric field pattern is as shown in my Fig. 4 until charge begins to flow. When this charge begins to build up on the insulated region between the ionizers, however, the electric field lines will then terminate on the ionizers only. Thus, the "number" of field lines serving as charge paths to the amplifier is essentially unchanged. I chose not to elaborate on this point in the interest of simplicity.

The first comment, however, concerning the ion mobilities is a valid one and does raise a serious objection to the model as proposed. I feel that their model presents a picture of this phenomenon which is much closer to reality.

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Comment on "A Criterion for Assessing Wind-Tunnel Wall Interference at Mach 1"

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GOODMAN[1] proposes a criterion for determining conditions for negligible wall interference in a sonic stream. The criterion that the tunnel similarity parameter G must be order one or greater for negligible interference is mathematically and physically unrealistic. The analysis contains several oversimplifications which are commented on here. Equation numbers and notation are those used in Goodman's Note.

Goodman's analysis adopts the transonic small disturbance equation for $M_\infty = 1$. The local linearization approximation is used to reduce the non-linear mixed elliptic-hyperbolic equation to a linear parabolic equation. This simplification results by assuming that the local flow acceleration $(\gamma + 1) U_X/U_\infty$ is a positive number λ throughout the flowfield. The flow problem is then likened to a one-dimensional heat conduction problem with the streamwise direction x becoming the time-like variable and the lateral direction z the space-like variable. The semi-infinite problem $0 \leq z \leq \infty$ corresponding to unbounded flow past an airfoil is considered. Body boundary conditions are applied at $z = 0$, and all disturbances are assumed to vanish at $z = \infty$.

The analysis is oversimplified in at least two respects. First, the assumption that the acceleration λ is positive throughout the flow field is erroneous. In fact λ may be positive, zero, or negative. The heat conduction problem as stated is ill-posed since the diffusity "constant" λ^{-1} changes sign. Thus, the relation following Eq. (4), $z \geq O(x/\lambda)^{1/2}$ which forms the basis for the criterion is impractical. Second, the wind-tunnel wall boundary condition is not considered in the analysis. The tunnel case corresponds to a heat conduction problem in a bounded domain $0 \leq z \leq H$ with appropriate conditions specified at both boundaries. As such, a solid wall would correspond to zero heat transfer ($\phi_z = 0$), a freejet to a constant temperature boundary (ϕ = constant), and ventilated walls to

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a combination of these. The nature of the solution will depend strongly on these boundary conditions.

Goodman introduces the similarity parameter G which is defined for two-dimensional [Eq. (8)] and axisymmetric [Eq. (14)] flow. These parameters are well known[2,3] and may be derived from the governing equations and boundary conditions without resorting to the local linearization approximation. The paradox which Goodman discusses has been explained by Berndt.[3] Goodman's definition of G for axisymmetric flow, Eq. (14), incorrectly contains a logrithmic term $(\ln[\tau^2(\gamma + 1)])^{1/2}$. The logrithmic factor in Eq. (11) enters only for values of U near the body due to the inner solution of slender body theory.[4] In the outer flow, which is where the transonic effects enter, the logrithmic term is inapplicable and should be dropped from Eq. (14).[3]

From a physical point of view it seems quite plausible that "the larger the value of G the less the wall interference will be." However, the statement that "For those tests for which G become of order one or greater, the interference may be presumed to be negligible" is unreasonable. For example, consider a sonic flow in a solid wall wind tunnel with no body present. The introduction of any body will choke the tunnel and reduce the freestream Mach number. The interference is far from negligible and the results must be interpreted carefully.[2] The experiments Goodman considers were all performed in transonic wind tunnels with ventilated test section walls that, presumably, were carefully developed with minimized wall interference in mind.

In conclusion, we feel that Goodman's analysis is oversimplified and that the conclusion that $G \geq 0(1)$ for negligible wall interference is misleading. The better conclusion is that in the limit $G \to \infty$, the wall interference goes to zero. Much of the previous work both on transonic scaling laws and development of transonic tunnel wall properties for minimizing interference effects is not recognized by the author. This work shows that the interference effects at Mach one are severe and complicated.

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Reply by Author to E. M. Murman and F. W. Steinle Jr.

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THE preceding Comment actually consists of several comments, and I should like to take them up one at a time.

It is true that the acceleration λ is assumed to be positive in deriving the criterion. However, for sonic flow there are many situations for which λ is positive in the field at stations near the foil, and for those cases for which λ reverses sign the heat conduction analog is still valid provided x is measured forward from the trailing edge. Since x is of the order of the airfoil chord in either case, the parameter G still falls out as the one by which wall interference should be judged.

With regard to the boundary condition at the wall, since the field equation is likened to the heat conduction equation, with x the time-like variable, it is appropriate to use an integral method to solve it,[1] in which case the boundary condition at the wall only comes into play when the penetration depth reaches the wall. If this should occur downstream of the trailing edge, the wall boundary condition will have little or no affect on the flow at the foil. Indeed, the criterion $G \geq 0(1)$ can be derived from just such considerations, and has been in Ref. 6 of the Note.

References 2 and 3 of the Comment were unfamiliar to me, but the fact that the similarity parameter G has been derived previously, without resort to the method of local linearization, lends support to the validity of this parameter and also to the method of local linearization.

I agree that the logarithmic term should be dropped from Eq. (14) and apologize for this error. Clearly the qualitative differences between two-dimensional and axially symmetric flow, as pointed out in the Note, are even greater without the logarithmic term.

With regard to choked flow, I was very careful to point out that the freestream Mach number must be unity so that ventilated walls were implied. In Ref. 6 of the Note closed walls are specifically excluded, but I believe that as long as the tunnel walls are ventilated in order to achieve a free stream Mach number of unity, and as long as the condition $G > G_{crit}$ is satisfied, the wall interference will be small.

Although it is ture that mathematically the interference goes to zero as $G \to \infty$, from the physical point of view $G \geq 0(1)$ is correct. The situation is similar to one encountered in boundary-layer theory. Strictly speaking the boundary condition in the freestream is to be applied at infinity in boundary-layer coordinates, but practically speaking little error is encountered if, instead, a momentum integral approach is used and the free-stream boundary condition is applied at the edge of the boundary layer. In Ref. 6 of the Note the integral approach leads to the condition $G > G_{crit}$ and also to numerical values of G_{crit} for several configurations.

Although Refs. 2 and 3 of the Comment were unfamiliar to me, as I have said, work on the development of transonic wall properties is certainly not. Indeed, I was the first to present the appropriate boundary condition for perforated walls and to derive a condition for zero blockage interference at subsonic speeds in a perforated wall tunnel.[2] This work has never been published in the open literature, but results contained in it are quoted in Ref. 3. The reason this work (and other work on slotted walls) was not "recognized" is because it is my belief that the wall boundary condition is irrevelant provided the walls are ventilated in order to achieve Mach one, and provided $G > G_{crit}$.

Finally, I should like to take this opportunity to add that it can be shown that the same condition as derived in the Note is applicable for thickness-dominated lifting configurations.

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